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ARMY MATERIEL SYSTEMS ANALYSIS AGENCY

TECHNICAL MEMORANDUM NO. 11

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ON A MULTIPLE EXPONENTIAL CHANNEL SERVICE FACILITY WITH HETEROGENEOUS MEAN SERVICE RATES

Michael G. Kolifrath
Arthur L. Smith

Mobility, Communication & Electronics Division

RDT&E Project No. 1T050222M603

ABERDEEN PROVING GROUND, MARYLAND

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MGKolifrath/ALSmith/mdh
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ABSTRACT

This report extends the "classical" model of a service facility with a given number of independent parallel channels and a single queue. The input to the system is Poisson, service times for all channels are exponentially distributed, and service is provided on a first-come, first-served basis. The extension is in the assumption that the channels need not have the same mean service rate.

The steady-state probability distribution of the number of units in the system is found. It is expressed, however, in terms of conditional probabilities that specific channels are occupied given a certain number in the system. These conditional probabilities are then found using a second formulation which explicitly defines all possible states of the system (both the number of units and where they are located).

TABLE OF CONTENTS

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I. INTRODUCTION

Consider a service facility with multiple independent channels in parallel, fed by a single queue with Poisson arrivals. The service times for all channels are exponentially distributed, but the mean service rates need not be the same. Units are serviced on a first-come, first-served basis.

This model is descriptive of a class of problems where we wish to construct a service facility from a finite population of servers having different service rates, so as to minimize the sum of the cost of providing service and the cost of waiting.

In the case of such a facility where the mean service rates are equal, the state of the system is explicitly defined by the number of customers in the system (both in service and in the queue). This is not always true in the model we are considering, for if the number of customers in the system is less than the number of channels, the probability that a service is completed in some small increment of time depends on which channels are occupied.

We will develop the steady-state probability distribution of the number of customers in the system, first for the two channel case, and then for a general number of channels.

II. DEFINITIONS AND ASSUMPTIONS

Let n = the number of customers in the system.

s = the number of service channels.

λ = the mean arrival rate of customers into the system.

μ_i = the mean service rate of channel i.

$P_n(t)$ = the probability that there are n customers in the system at time t. It is assumed that $P_n(t) = 0$ for $n<0$.